ACOR: Adiabatic Code of Oscillations including Rotation

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Collaborators

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Studying stellar oscillations with 2D complete models...

Why Bother ? !

Equations of stellar structure

Hydrodynamics equations, no viscous force, no tidal force, no magnetic field, Newtonian gravitation

Mass Conservation :

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\overrightarrow{v} = \frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\overrightarrow{v}) = 0$$

Momentum Conservation :

$$\frac{D\overrightarrow{v}}{Dt} = \frac{\partial\overrightarrow{v}}{\partial t} + \overrightarrow{v}\cdot\nabla\overrightarrow{v} = -\nabla\Phi - \frac{\nabla P}{\rho}$$

Poisson Equation for the gravitational potential :

$$\nabla^2\Phi = 4\pi G\rho$$

Energy Conservation :

$$T\frac{Ds}{Dt} = \frac{Du}{Dt} + P\frac{DV}{Dt} = \epsilon - \frac{\nabla\cdot\overrightarrow{F}}{\rho}$$

as well as energy transport equation which allows to determine $\overrightarrow{F}$.

Oscillation equations

$\tau_{osc} \approx \tau_{dyn} \ll \tau_{nuc}$
→ Perturbation around a stationnary, axis-symmetric equilibrium state

$\tau_{osc} \approx \tau_{dyn} \ll \tau_{KH}$
→ Adiabatic perturbation in most of the stellar interior

Continuity equation

$$\rho' + \vec{\nabla}\cdot(\rho_0\vec{\xi}) = 0$$

Momentum equation

$$\left[\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\varphi}\right)v'_i\right]\mathbf{e_i} + 2\Omega\times\mathbf{v}' + \left(\mathbf{v}'.\nabla\Omega\right)r\sin\theta\,\mathbf{e}_\varphi = -\frac{1}{\rho_0}\nabla p' - \nabla\Phi' + \frac{\rho'}{\rho_0^2}\nabla p_0$$

Adiabatic relation

$$\left(\frac{\partial}{\partial \mathrm{t}} + \mathbf{v_0}\cdot\nabla\right)\left(\frac{\rho'}{\rho_0} - \frac{p'}{\Gamma_1 p_0}\right) + \frac{\partial\xi}{\partial \mathrm{t}}.\left(\frac{\nabla\rho_0}{\rho_0} - \frac{\nabla p_0}{\Gamma_1 p_0}\right) = 0$$

Poisson Equation

$$\nabla^2\Phi' = 4\pi \mathrm{G}\rho'$$

Coordinate system

Evolved stellar models ⇒ Abrupt variations occuring on isobars

Inspired from Bonazzola et al. (1998), Reese et al. (2006)

Multi-domain method

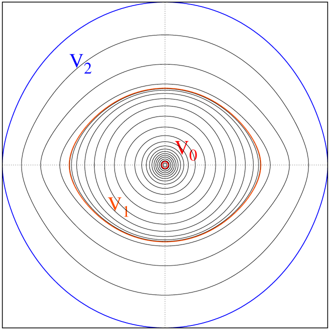
- ⋆ $\theta, \varphi \rightarrow$ spherical system
- ⋆ new radial coordinate : ζ

V_0 : from a spherical $\zeta \rightarrow \zeta$ constant on isobars

V_1 : ζ constant on isobars

V_2 : from ζ constant on isobars $\rightarrow$ spherical ζ

2 $\mathrm{M}_\odot$, $X_c = 0.35$, $\Omega = 80\%\,\Omega_k$



⇒ Benefit : simplifies greatly boundary conditions
+ Avoids dirty radial derivatives (at convective-radiative transitions)

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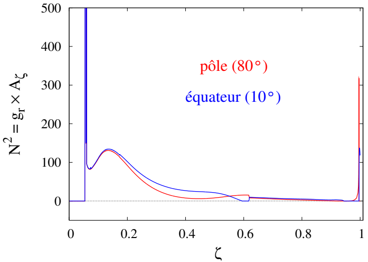

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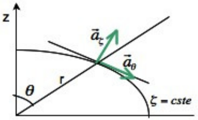
Expansion in series of spherical harmonics

- ⋆ Spheroïdal vector basis $(\vec{a}_\zeta, \vec{a}_\theta, \vec{e}_\varphi)$ where :
  $\vec{v}' = v'_\zeta\,\vec{a}_\zeta + \tilde{v}'_\theta\,\vec{a}_\theta + v'_\varphi\,\vec{e}_\varphi$



- ⋆ v'_ζ and the scalar perturbations expended as series :

$$v'_\zeta = \sum_{\ell_2 \geq |m|}^{+\infty} u'_{\ell_2}(\zeta)\, Y_{\ell_2,m}(\theta,\varphi)$$

for the velocity vector horizontal components :

$$\tilde{v}'_\theta(\zeta,\theta,\varphi) = i \sum_{\ell_2 \geq |m|}^{+\infty} \left(\underbrace{v'_{\ell_2}(\zeta)}_{\text{poloidal}} \frac{\partial Y_{\ell_2,m}(\theta,\varphi)}{\partial\theta} + \underbrace{w'_{\ell_2}(\zeta)}_{\text{toroidal}} \frac{m}{\sin\theta} Y_{\ell_2,m}(\theta,\varphi) \right)$$

$$v'_\varphi(\zeta,\theta,\varphi) = -\sum_{\ell_2 \geq |m|}^{+\infty} \left(v'_{\ell_2}(\zeta) \frac{m}{\sin\theta} Y_{\ell_2,m}(\theta,\varphi) + w'_{\ell_2}(\zeta) \frac{\partial Y_{\ell_2,m}(\theta,\varphi)}{\partial\theta} \right)$$

Problem unknowns → components u'_{ℓ_2}, v'_{ℓ_2}, w'_{ℓ_2}, and P'_{ℓ_2}, ρ'_{ℓ_2}, Φ'_{ℓ_2}, $\partial_\zeta \Phi'_{\ell_2}$

Expansion in series of spherical harmonics

→ The equation system is projected on the eigenspace relative to $Y_{\ell_1,m}$:

$$\forall \ell_1 \geq | m |, \qquad \sum_{\ell_2 \geq |m|}^{+\infty} \int \frac{\sin\theta \mathrm{d}\theta \mathrm{d}\varphi}{4\pi} \mathrm{E}^{\ell_2}(\zeta,\theta) \mathrm{Y}_{\ell_2}^{\mathrm{m}} \mathrm{Y}_{\ell_1}^{\mathrm{m}*} = 0$$

Given the equatorial symmetry of the model of structure, and the properties of $Y_{\ell,m}$

⇒ Selection rules : Only the Y_ℓ^m of same parity are selected in the expansion
e.g. : for an even mode with $m = 1 \rightarrow \ell = 1, 3, 5, 7, \cdots$

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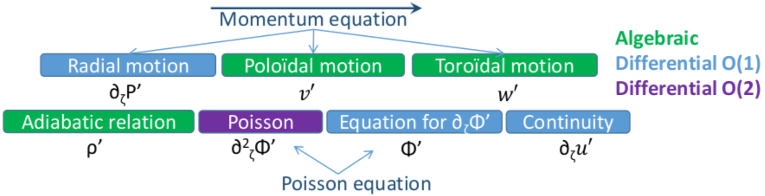

⇒ the system is reduced to 4 ODEs with P', Φ', $\partial_\zeta \Phi'$, and u'

Boundary value problem

⋆ At the center : $\zeta = 0$

We impose a regularity condition :

For scalar unknowns : $\Phi'_\ell = \mathcal{O}\left(\zeta^\ell\right)$

For the velocity components :

$$\forall \ell \neq 0 \;\; u'_\ell = \mathcal{O}\left(\zeta^{\ell-1}\right), v'_\ell = \mathcal{O}\left(\zeta^{\ell-1}\right) \text{ and } w'_{\ell_p} = \mathcal{O}\left(\zeta^{\ell_p}\right)$$

$$\text{for } \ell = 0 : \; u'_0 = \mathcal{O}(\zeta) \text{ and } v'_0 = 0$$

⋆ At the surface : $\zeta = 1$

Stress-free condition : $\delta P = 0$

⋆ On the outer surface (domain V_2) : $\zeta = 2$

The gravitational potential goes toward 0 at infinity :

$$\mathrm{d}\Phi'_\ell = -(\ell+1)\frac{\partial_\zeta \mathrm{r}}{\mathrm{r}}\Phi'_\ell$$

Finite differences for radial resolution

The problem can be put under the form :

$$\forall\zeta \quad \frac{\mathrm{d}y}{\mathrm{d}\zeta} = \tilde{\mathrm{A}}\, y$$

y : containing the spectral components of the 4 unknowns

Radial Discretization à la Scuflaire et al. (2008)

$$y_i + \frac{h}{2}\left.\frac{\mathrm{d}y}{\mathrm{d}\zeta}\right)_i + \frac{h^2}{12}\left.\frac{\mathrm{d}^2 y}{\mathrm{d}\zeta^2}\right)_i = y_{i+1} - \frac{h}{2}\left.\frac{\mathrm{d}y}{\mathrm{d}\zeta}\right)_{i+1} + \frac{h^2}{12}\left.\frac{\mathrm{d}^2 y}{\mathrm{d}\zeta^2}\right)_{i+1} + o\left(h^5\right)$$

$$\left[\mathrm{I_d} + \frac{\mathrm{h}}{2}\tilde{\mathrm{A}}_{\mathrm{i}} + \frac{\mathrm{h}^2}{12}\left(\tilde{\mathrm{A}}_{\mathrm{i}}^2 + \left.\frac{\mathrm{d}\tilde{\mathrm{A}}}{\mathrm{d}\zeta}\right|_{\mathrm{i}}\right)\right] y_i =$$

$$\left[\mathrm{I_d} - \frac{\mathrm{h}}{2}\tilde{\mathrm{A}}_{\mathrm{i+1}} + \frac{\mathrm{h}^2}{12}\left(\tilde{\mathrm{A}}_{\mathrm{i+1}}^2 + \left.\frac{\mathrm{d}\tilde{\mathrm{A}}}{\mathrm{d}\zeta}\right|_{\mathrm{i+1}}\right)\right] y_{i+1} + o\left(\mathrm{h}^5\right)$$

⇒ Benefits

precise : to the 5th order in terms of the radial resolution

stable : discretization over 2 successive layers only

→ Point of attention

requires to differentiate radially the model of structure

Buidling up the matrices

Trial value σ_0, seeking $\delta\sigma$ with $\sigma = \sigma_0 + \delta\sigma$

$$\mathrm{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \quad \text{(layers 1, } \cdots \text{ N)} \qquad \mathscr{A}\,\mathrm{Y} = \delta\sigma\,\mathscr{A}_{\delta\sigma}\,\mathrm{Y}$$

Eigenvalue problem resolution, and iteration $\sigma_0 = \sigma_0 + \delta\sigma$ until convergence

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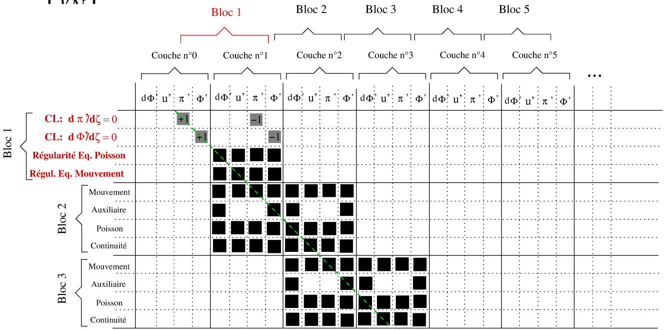


e.g. : when 26 spherical harmonics are used for the expansion, 2000 points in radius $\rightarrow$ 4.6×10^7 non null terms, for 5.5×10^{10} elements

Eigenvalue problem resolution, and iteration $\sigma_0 = \sigma_0 + \delta\sigma$ until convergence
$\rightarrow$ **Generalized inverse iteration method** (Dupret et al. 2001)

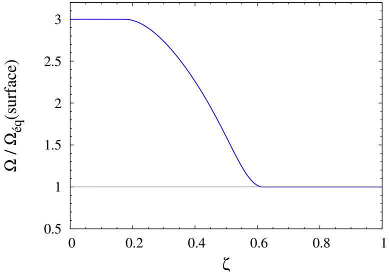
Results

Pulsations of a rapidly rotating intermediate-mass star

Pulsational study of self-consistently distorted non-homogeneous stellar models (Roxburgh et al. 2008)

An example : $2M_\odot$, $X_c = 0.35$, $\Omega(r)$ (non barotropic) with STAROX-2D

Flatness : $1 - \mathrm{R_{pol}}/\mathrm{R_{eq}} \simeq 0.25$; $V_{eq,s} = 282.3\,\mathrm{km.s^{-1}}$, $\Omega_{s,eq}\sqrt{R_{eq}^3/GM} = 0.80$.



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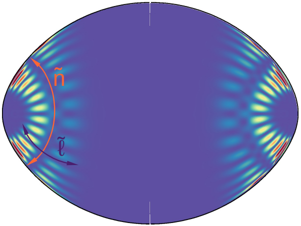

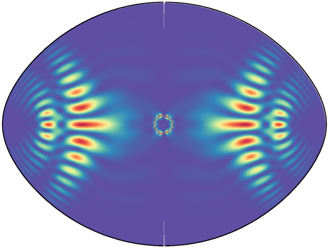

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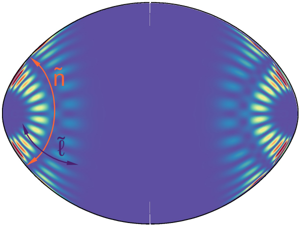

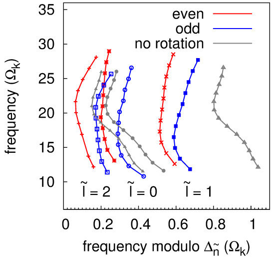

⇒ Towards interpretation of δScuti stars prolific seismic spectra

Near degeneracies in rotationally splitted mixed modes

$\ell = 2$ rotationaly splitted modes in avoided crossing (Deheuvels+ 2017)

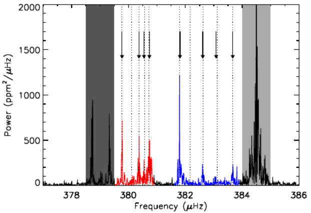

Fig. 1. Section of the oscillation spectrum of KIC 7341231 in the neighborhood of the 384-μHz radial mode (light gray area). The dark gray area corresponds to a rotationally-split $l = 1$ mode and the vertical arrows indicate the observed mode frequencies of two $l = 2$ mixed modes (see Table 1).

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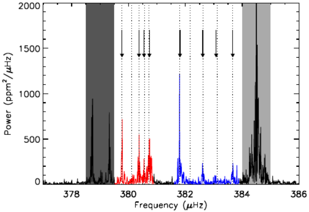

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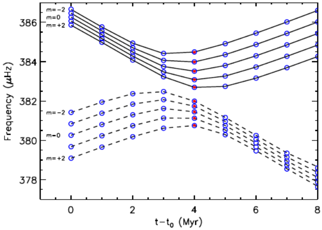


Near degeneracies in rotationally splitted mixed modes

$\ell = 2$ rotationaly splitted modes in avoided crossing (Deheuvels+ 2017)

Perturbative approach :

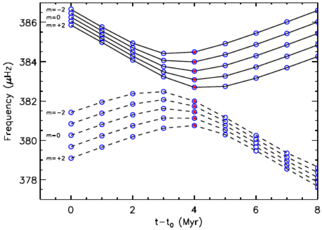
$(\mathscr{L}_0 + \mathscr{L}_1)\xi - \omega^2\xi = 0$, where

$\mathscr{L}_0\xi = \dfrac{\nabla P'}{\rho_0} - g' - \dfrac{\rho'}{\rho_0} g_0$, and

$\mathscr{L}_1\xi = 2\omega(m\Omega - i\Omega\times)\xi$

Then $\omega_1 = \dfrac{1}{2\omega_0} < \mathscr{L}_1\xi_0|\xi_0 >$, hence

$\omega_1 = m\int_0^R K(r)\Omega(r)dr$



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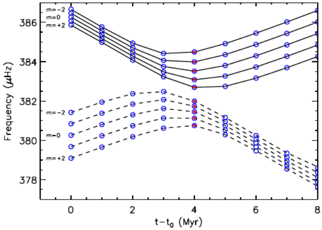
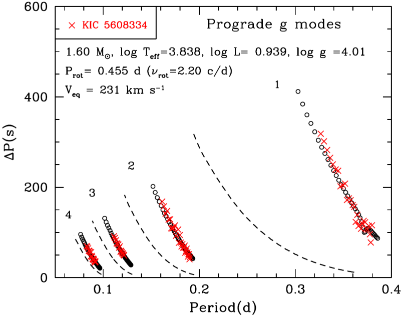


Modes of different azimutal order m avoid crossing at different evolutionnary times.

$\Rightarrow$ Only a method which solves the ξ_m eigenmodes individually can capture this.

Pure inertial modes in γ Doradus stars

Saio et al. 2018



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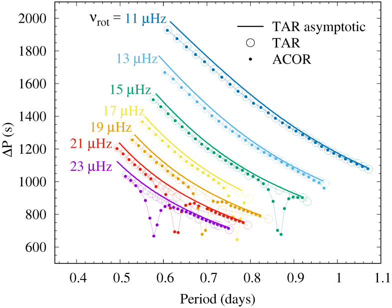

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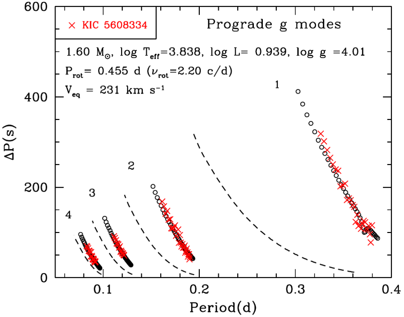
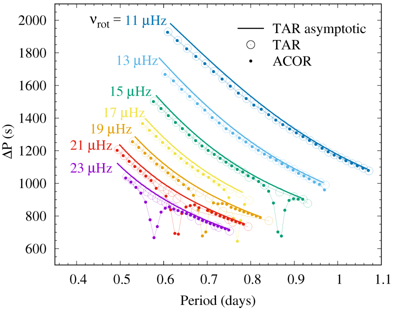
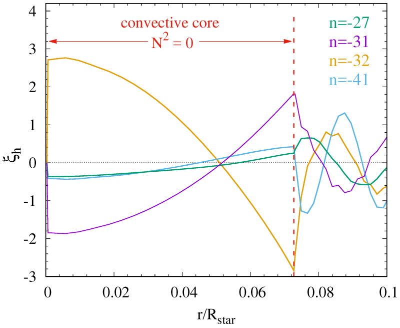


For uniformly rotating models:

- Complete non-perturbative calculations ACOR, Ouazzani et al. 2012, 2015
- TAR implemented in the LOSC code Scuflaire et al. 2008
- TAR asymptotic Townsend 2003b $P_{co} = P_0(n+\epsilon)/\sqrt{\lambda_{k,m}(s)}$

Pure inertial modes in γ Doradus stars





→ Hypothesis of strong stratification of the TAR ($\Omega \ll N$) not valid ?

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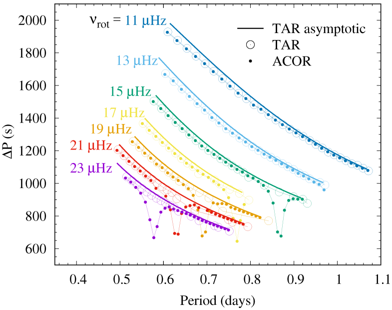

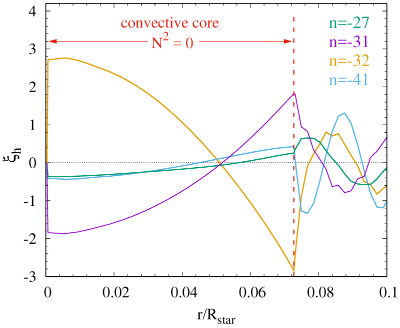

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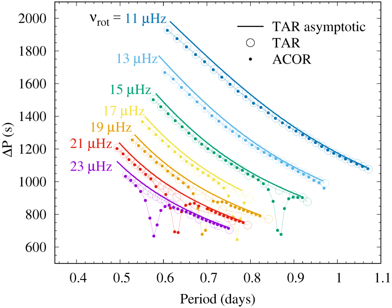
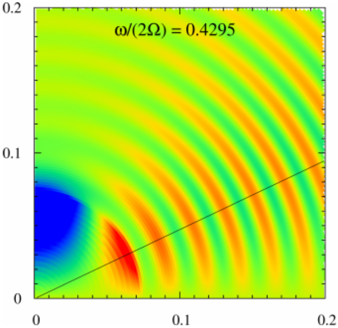
Could there be coupling with pure inertial modes ?

- modes which restoring force is the Coriolis force only,
- and which can propagate in regions where buoyancy is zero (ie convective).

Pure inertial modes in γ Doradus stars





→ Signature of the resonances between pure inertial modes and gravity modes Ouazzani et al. 2020

Compatibility with evolution codes

The ACOR code can interface with...

⋆ CESTAM (Lebreton & Morel 2008, Marques et al. 2013)
→ In particular with CESTAM−2D : (Manchon et al. in prep)
same coordinate system (Ouazzani+ 2015) ⇒ no extra-differentiation

⋆ CLES (Code Liégeois d'Évolution Stellaire)
1D evolution code, particularly well designed to follow evolution of convective cores in intermediate-to-massive stars.
→ In particular it uses double mesh points at the convective-radiative interface.
In absence of mixing, chemical discontinuity for cores that are growing on the MS.
To avoid impact on the period spacing of g-modes
→ double mesh point, treated in ACOR with a matching condition (Ouazzani+ 2017).

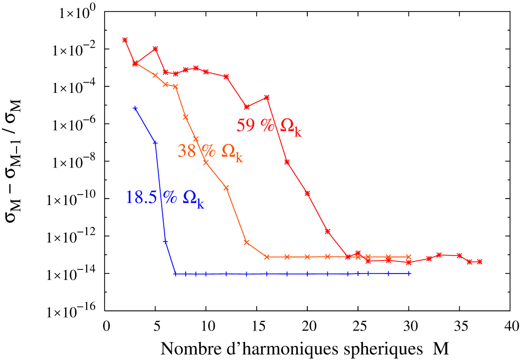
⋆ STAROX-2D (Roxburgh et al. 2008) same as CESTAM−2D

Also takes : *.FGONG, and *.AMDL.

Additional material

Performances : spectral resolution

⋆ Convergence as a function of the number of Y_ℓ^m used for the spectral expansion



$$\left.\frac{\delta\sigma}{\sigma}\right)_{CoRoT} \sim 10^{-4} - 10^{-3}$$

Polytropic models
2 000 points in radius

⋆ Convergence in terms of radial resolution : $\rightarrow \delta\sigma/\sigma \sim 10^{-6}$

Generalized inverse iteration method

Théorème :

pour le problème aux valeurs propres généralisé $\mathscr{A}\,\mathrm{Y} = \delta\sigma\,\mathscr{A}_{\delta\sigma}\,\mathrm{Y}$,
$\mathscr{A}$ et $\mathscr{A}_{\delta\sigma}$ matrices carrées de dimension N telles que :

- $\mathscr{A}$ est inversible
- $\mathscr{A}^{-1}\mathscr{A}_{\delta\sigma}$ est diagonalisable

Alors pour tout vecteur propre initial Y_0 et pour tout scalaire $\delta\sigma_0$,

$$\mathrm{Y}_{\mathrm{k+1}} = (\mathscr{A} - \delta\sigma_0\,\mathscr{A}_{\delta\sigma})^{-1}\,\mathscr{A}_{\delta\sigma}\,\mathrm{Y}_{\mathrm{k}}$$

converge géométriquement vers le vecteur propre du système dont la valeur propre est la plus proche de $\delta\sigma_0$.

valeur propre donnée par : $$\delta\sigma = \frac{\mathrm{Y}^*\,\mathscr{A}^*_{\delta\sigma}\,\mathscr{A}\,\mathrm{Y}}{\mathrm{Y}^*\,\mathscr{A}^*_{\delta\sigma}\,\mathscr{A}_{\delta\sigma}\,\mathrm{Y}}$$